Reading Assignment 5 (Due Wednesday 7/7/21 by 12:55 PM)

Basic learning objectives: These are the tasks you should be able to perform with reasonable fluency **when you arrive at our next class meeting**. Important new vocabulary words are indicated in italics.

1. Compute integrals of vector-valued functions using the rules of integration you learned in single-variable calculus.
2. Compute *limits* of multi-variable functions using Properties of Limits. Use appropriate methods to prove that a limit does not exist.
3. Determine whether a function is continuous at a point using various methods.
4. Compute *partial derivatives* of multivariable functions using only the limit definition.

Advanced learning objectives: In addition to mastering the basic objectives, here are the tasks you should be able to perform **after class, with sufficient practice:**

1. Understand how the level curves of a multivariable function can provide insight into whether a limit does or does not exist.
2. Compute partial derivatives using the differentiation laws from calculus of a single variable.
3. Interpret the first partial derivatives $f_x(x, y)$ and $f_y(x, y)$ geometrically in relation to the graph of the function. Interpret the first partial derivatives as a rate of change.
4. Estimate partial derivatives given a table of finitely many values of a function, or a contour plot of a function.

Directions: Read the following sections of the book:

- Sections 9.7.1, 9.7.2, 9.7.3, and 9.7.4. Optional: Section 9.7.5 (you won't need to know this for exams).
- All of Section 10.1.
- Section 10.2.1 up to and including Definition 10.2.4.

and complete the following tasks along the way. If an Activity is not listed, you do not need to complete it (although you are welcome to read it). Turn your write up in via gradescope. You do not need to write the questions down, as long as you clearly indicate the question number.

1. Activity 9.7.6.

2. Preview Activity 10.1.1.

3. Activity 10.1.2.

4. Activity 10.1.3.

5. Compute the following limits using the properties of limits. Justify your reasoning. Compare with the associated graph.

(a) $\lim_{(x,y)\to(\pi,\pi/2)} y\sin(x-y)$. The graph: GeoGebra: Reading 5 Task 4(a).

(b) $\lim_{(x,y)\to(0,0)} \dfrac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}$. The graph: GeoGebra: Reading 5 Task 4(b).

6. Use appropriate limit along different paths to show that the following limits do not exist. Justify your reasoning. Compare with the associated graph.

(a) $\lim_{(x,y)\to(0,0)} \dfrac{xy^4}{x^2+y^8}$. GeoGebra can't accurately graph this one near the limit.

(b) $\lim_{(x,y)\to(0,0)} f(x,y) = \dfrac{3x^2}{3x^2+5y^2}$. The graph: GeoGebra: Reading 5 Task 5(b)

7. Use the properties of continuity to determined the set of points for which the following functions are continuous. Justify your reasoning. Compare with the included graph.

(a) $f(x,y) = \dfrac{1}{9-x^2-y^2}$. The graph: GeoGebra: Reading 5 Task 6a. Note: computer generated graphs exhibit strange behavior near some discontinuities!

(b) $f(x,y) = \dfrac{e^x+e^y}{e^{xy}-1}$. The graph: GeoGebra: Reading 5 Task 6b.

8. After reading all of Section 10.1, write down three things you learned or still have questions about. **I will not be lecturing on 10.1, but we will do a reading debrief.**

9. Preview Activity 10.2.1.

10. Use Definition 10.2.4 to compute $f_x(x,y)$ and $f_y(x,y)$ for the following functions:

(a) $f(x,y) = xy^2 - x^3y$.

(b) $f(x,y) = \dfrac{x}{x+y^2}$.

The correct answers are included in the footnote. [1]

[1] 5(a) $\lim_{(x,y)\to(\pi,\pi/2)} y\sin(x-y) = \pi/2$, 5(b) $\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} = 2$
6(a) approach $(0,0)$ along the paths $r_1(t) = (t,0)$ (the x-axis) and $r_2(t) = (t, t^{1/4})$ (the graph of $f(x) = x^{1/4}$ in the x,y-plane).
6(b) approach $(0,0)$ along the x-axis and the y-axis.
7(a) The circle of radius 3 centered at the origin. 7(b) every point in $\mathbb{R}^2$ except points on the lines $x=0$ and $y=0$.
9(a) $f_x(x,y) = y^2 - 3x^2y,\ f_y(x,y) = 2xy - x^3$ 9(b) $f_x(x,y) = \frac{y^2}{(x+y^2)^2},\ f_y(x,y) = -\frac{2xy}{(x+y^2)^2}$